



Exploring the Predictive Power of Problem-Solving Skills, Conceptual Understanding, and Logical Reasoning on Mathematics Achievement

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ABSTRACT: The purpose of the study was to explore the Predictive Power of Problem-Solving Skills, Conceptual Understanding, and Logical Reasoning on Mathematics Achievement. The objectives of the study were to assess the effect of problem-solving skills on mathematics achievement, to assess the effect of conceptual understanding on mathematics achievement, and to assess the effect of logical reasoning on mathematics achievement. The study employed a descriptive research design utilizing a quantitative approach. The population of the study consists of senior high school students. The target population for the study comprises all students within Atwima Nwabiagya Municipality. The accessible population was 2528 students. The study employed a convenience sampling technique to select the schools. Again, simple random sampling techniques were used to select students from the selected schools. A sample of 345 students was used for the study. Structured questionnaires with closed-ended questions were used as the main instrument to collect data from the respondents. A structural Equation Modeling (SEM) via Amos (ver. 23) was used to analyze the data collected to answer the research hypothesis. The study revealed that Problem-Solving Skills (PSS) had a significant positive effect on Mathematics Achievement (MA). Again, the study also revealed that Conceptual Understanding (CU) had a positive significant effect on mathematics achievement (MA). Also, the results indicated that Logical Reasoning (LR) positively influenced Mathematics Achievement (MA). This study contributes to existing literature by exploring the combined effect of problem-solving skills, conceptual understanding, and logical reasoning on mathematical achievement among senior high school students.

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KEYWORDS:

Problem-Solving Skills, Conceptual Understanding, Logical Reasoning, Mathematics Achievement

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INTRODUCTION

Mathematics plays a crucial role in fostering intellectual growth, technological advancement, and national development. It equips learners with logical thinking, problem-solving, and analytical skills necessary for daily life and professional success. Tashtoush and Qasimi, (2024), mathematics achievement serves as an essential benchmark of academic quality and economic progress in both developed and developing nations. Despite its significance, poor performance in mathematics continues to be a global concern, especially in developing countries such as Ghana, where students consistently perform below expectations in national and international assessments (Akpalu et al., 2025). This persistent issue underscores the need to explore the underlying cognitive and metacognitive factors influencing students' success in mathematics.

Among the various factors contributing to students' mathematics achievement, cognitive skills such as problem-solving, conceptual understanding, and logical reasoning have been identified as key predictors of academic performance. Problem-solving skills enable students to analyze mathematical problems, develop solution strategies, and apply relevant concepts effectively (Roorda et al., 2024). Conceptual understanding, on the other hand, reflects a learner's ability to comprehend the relationships among mathematical ideas and to apply them flexibly in different contexts (Ncube et al., 2024). Logical reasoning supports the process of drawing valid

conclusions and establishing connections between mathematical concepts, allowing students to approach problems systematically and make sound judgments (Säfström et al., 2024).

Despite these critical cognitive abilities, there remains a gap in understanding how these variables jointly predict mathematics achievement, particularly among secondary school students in Ghana. Previous research indicates that many Ghanaian learners tend to rely heavily on rote memorization and procedural learning rather than deep conceptual understanding or analytical reasoning (Larbi & Vimolan, 2025). This overreliance on procedural methods often leads to difficulties in tackling complex or unfamiliar problems that require higher-order reasoning and creative thinking (Asare, 2026). Consequently, investigating how problem-solving skills, conceptual understanding, and logical reasoning interact to influence mathematics achievement can provide valuable insights into improving learning outcomes (Bright Asare, 2024).

Furthermore, international reports such as the Programme for International Student Assessment (PISA) emphasize that the mastery of reasoning and problem-solving abilities is essential for lifelong learning and adaptation to modern challenges (Alali & Wardat, 2024). These skills not only enhance mathematical competence but also contribute to the development of critical thinking and decision-making abilities that extend beyond the classroom. Understanding the predictive relationship between these cognitive skills and mathematics achievement can therefore guide educators, curriculum planners, and policymakers in designing effective instructional approaches that foster deep learning and improved student outcomes (Sliwka et al., 2026). In light of these considerations, this study seeks to explore the predictive power of problem-solving skills, conceptual understanding, and logical reasoning on mathematics achievement among secondary school students. By examining how each of these factors contributes to mathematics performance, the study aims to provide empirical evidence that can inform teaching practices and curriculum reforms. The findings are expected to support strategies that not only enhance students' academic success in mathematics but also cultivate critical and logical thinking skills necessary for 21st-century learning.

Research Objectives

1. To assess the effect of problem-solving skills on mathematics achievement.
2. To assess the effect of conceptual understanding on mathematics achievement.
3. To assess the effect of logical reasoning on mathematics achievement.

Research Hypothesis

H₁: Problem-solving skills positively predict mathematics achievement.

H₂: Conceptual understanding positively predicts mathematics achievement.

H₃: Logical reasoning positively predicts mathematics achievement.

LITERATURE REVIEW

Problem-Solving Skills and Mathematics Achievement.

A study conducted by Bushra Naz and Aminah Qayyum (2025) effectiveness of problem-solving as a teaching strategy in enhancing students' conceptual understanding and critical thinking in mathematics education. The study employed a quantitative, quasi-experimental study design. The two groups were the experimental group, which received instructions using problem-solving strategies, and the control group, which received conventional lecture methods. The post-test measured the impact of the intervention, while the pre-test evaluated group equivalency. The population of the study consists of 379 secondary school students in the three districts of Southern Punjab, Pakistan, who attended both public and private schools. Since grade 9 mathematics students cover language that gives life to basic algebraic concepts that call for strong mathematical concepts and analytical knowledge, this group of students was chosen. Statistical package for the Social Sciences (SPSS) ver. 26 was used to conduct the statistical analysis. The experimental and control groups' post-test results were measured to evaluate the problem-solving-based approach, and independent samples t-tests were also used to compare the means. Using paired sample t-tests, within-group improvements from the pre-test to the post-test were examined. The relationship between concept understanding and critical thinking of students was determined using Pearson's correlation, but separately for each test.

Again, Adeoye and Jimoh (2023), conducted a study on problem-solving skills among 21st-century learners towards creativity and innovation ideas. The study used a systematic literature review by identifying, evaluating, and interpreting available research that is relevant to the formulation of the problem and the topic area studied. The study collected journals from Google Scholar, Research Gate, SINTA, Scopus, and Web of Science. The study employed a CPS model, which consists of six stages, namely, 1) understanding the problem, 2) generating ideas, 3) developing ideas, 4) developing solutions, 5) planning for action, and 6) evaluating results. The study found that problem-solving skills enable learners to analyze complex problems, develop creative solutions, and implement those solutions effectively.

Moreover, a study by Harjati et al. (2025) on the mathematics problem-solving abilities of Indonesian school students in Sekolah Indosea Luar Negeri (SILN) based on Polya's Theory in terms of the pragmatic. The study employed a descriptive qualitative approach to explore the learning difficulties faced by Indonesian school students in Sekolah Indonesia Luar Negeri (SILN), particularly in the topic of Two-Variable Linear Equation Systems (SPLDV). The study focuses on the Four Stages of Polya for

effective problem solving, namely 1) understanding the problem, 2) devising a plan, 3) carrying out the plan, and 4) evaluating the solution. The study employed a snowball sampling technique during the research process. The data collection methods include a problem-solving test on systems of linear equations in two variables (SPLDV), which was distributed via WhatsApp; a questionnaire administered through Google Forms, completed by 170 students; and an interview conducted with mathematics teachers and students via Zoom.

Furthermore, a study conducted by Sinaga et al., (2023) on the influence of students problem-solving understanding and results of students mathematics learning. The study population consisted of all students in fourth to sixth grade of primary schools in two states that were chosen randomly, with 82 students in grade IV, 84 students in grade V, and 97 students in grade VI. A sample of 263 students was used for the study. The instruments items used for data collection include a mathematics test, which consists of seven questions and questionnaire regarding mathematics problem-solving understanding, which consists of four questions. The study found that students' solving understanding has a significant influence on mathematics learning results, with a p-value of .000. *H₁: Problem-solving skills positively predict mathematics achievement.*

Conceptual Understanding and Mathematics Achievement.

A study conducted by Fauziyah and Hakim (2025) on analysis of students' mathematical conceptual understanding based on differences in mathematics thinking styles. The study employed a qualitative research method using interviews based on tasks through the think-aloud method for the thinking process in understanding mathematical concepts. The instruments used for data collection consist of a written test designed to probe students' conceptual understanding. The study utilized a time triangulation technique to ensure the validity of the research data. The data analysis followed a structure processes: data classification, reduction, presentation, interpretation, and conclusion drawing. The study emphasized three different mathematical thinking styles, namely, visual, analytical, and integrated, which shared similarly high mathematical ability levels. The study revealed that subjects with a visual mathematical thinking style create representations in the form of images, tables, or graphs for each indicator of understanding. Subjects with an analytical mathematical thinking style create representations in the form of verbal sentences or mathematical models that are rich in mathematical symbols and variables. While subjects with an integrated mathematical thinking style create representations in the form of verbal sentences, images, diagrams, mathematical symbols, tables, mathematical models, or graphs. The study concluded that while the type of mathematical thinking style- visual, analytical, or integrated-does not significantly influence the depths of students' understanding, it does shape the strategies they employ to construct that understanding.

Again, a study conducted by Reni et al. (2025) on the effect of ethnomathematics-based problem-based learning on students' conceptual understanding in mathematics. The study employed a quasi-experimental research design. This design was chosen for practical and ethical considerations, as administering a pretest was deemed likely to influence the learning process (testing effect) and potentially bias the post-test results. The population of the study consists of all eighth-grade students at MTs Putra AI-Ishlahuddiy Kediri, with a total of 193 students. The study employed a purposive sampling technique to select the students. Two classes were purposively selected: an experimental class with 20 students and a control class with 27 students, based on initial ability homogeneity and scheduling considerations. The instrument used was an essay test consisting of two items. The instrument underwent a content validation process conducted by mathematics education lecturers and subject teachers. The data analyzed involved pre-requisite tests (normality and homogeneity) and hypothesis testing using the independent samples t-test. The study revealed that ethnomathematics problem-based learning (PBL) enhances students' development of critical and logical thinking skills, as well as in connecting mathematical concepts with everyday contexts. The study found a statistically significant difference between the experiential and control groups ($p < 0.05$), indicating that the ethnomathematics-based PBL model positively influences students' conceptual understanding.

Also, a study conducted by Hussein (2023) on the effect of teaching conceptual knowledge on students' achievement, anxiety about, and attitude towards mathematics. The study employed a quasi-experimental research design. The purposive sampling technique was used to select 200 secondary school students from Erbil, Iraq. In the experimental group, conceptual teaching was the focus. In the control group, conventional teaching was used. Pre-test and post-test for the achievement test, mathematics attitude scale, and abbreviated math anxiety scale were applied to both groups to reveal the effect of conceptual knowledge on students' achievement, attitude, and anxiety, respectively. Repeated measure ANOVA was used to analyze the data. The study found that there is a statistically significant difference in mathematics achievement between the two groups ($p < .001$). Students' attitude towards mathematics in the treatment group developed positively, while teaching mathematics conceptually reduced anxiety among female students more effectively than it did among male students.

Furthermore, Mayasari (2021) study was conducted on the ability of students' conceptual understanding in completing story problems in mathematics. The study employed a descriptive analysis with a qualitative research approach. The study initially used the seventh-grade class of SMP Negeri 8 Merauke, which clearly showed that the students' conceptual understanding was still very low. Based on this assertion, the study then used 9 students as subjects who were divided into high, medium, and low categories. The instrument for data collection was a test of the ability to understand mathematical concepts, assessment rubrics, and test results of interviews with 9 research subjects. The study compiled a conceptual understanding problem consisting of 4 descriptive questions related to integers. These questions were in the form of problems that students often face in their daily lives. Again, the study also

compiled an assessment rubric that was developed in accordance with the indicators of understanding mathematical concepts. The study conducted interviews with 9 research subjects consisting of 3 high abilities, 3 medium abilities, and 3 low abilities. The data analysis techniques employed in the study were data reduction by analyzing data qualitatively by reducing data, presenting data, and drawing conclusions. The study revealed that student's ability at high ability could master the 6 indicators of concept understanding, students in the moderate category mastered 5 indicators of concept understanding, and students in the low category mastered 4 indicators of conceptual understanding.

H₂: Conceptual understanding positively predicts mathematics achievement.

Logical Reasoning and Mathematics Achievement.

Nawal et al. (2023) conducted a study on logical mathematical intelligence and its impact on the academic achievement of pre-service math teachers. The study adopted a descriptive analytical survey method. The study's population consisted of all pre-service female mathematics. A sample of 45 female students was used for the study. A comprehensive sample was chosen. The sample was in two folds: 1) an exploratory sample of 10 students to verify the psychometric properties of the questionnaires. 2) For the second part, 35 students were used to respond to the questionnaire after verifying its validity and reliability. The study used a structured questionnaire with closed-ended questions with a 5-point Likert scale. The study revealed that the general average of the logical mathematical intelligence level for the fourth year students was at a high level, when the arithmetic mean of the whole questionnaire was 3.71, standard deviation of 0.74. The marks of students' semester average were: Agree with a rate of 36.4%, strongly agree with a rate of 29.5%. The cumulative average was 72.7% for a good level, and 25% for a very good level. The study finally revealed that there is a significant effect between the logical mathematical intelligence of the female students and the overall academic achievements in mathematics.

Furthermore, Ramganes and Reddy (2021), logical reasoning of school students as a predictor of their academic performance in mathematics. The study employed a descriptive research survey design. A sample of 540 standard ninth-grade students in the Kanchipuram region from 13 high/higher secondary schools was selected through a stratified random sampling technique. The tools developed by the investigators were the Logical Reasoning questionnaire for High School Students (LRQHSS), consisting of 25 items, despite being considered the half-yearly examination, math marks of students as an indicator of academic performance. The study utilizes internal consistency, such as a measure of reliability; the split-half method of reliability was employed. The reliability coefficient of the Logical Reasoning Questionnaire for High School Students (LRQHSS) was 0.759. The study revealed that as many as 67.8% of the variance could be predicted from students' Logical Reasoning on Academic Performance of students in mathematics, and a significant correlation between logical reasoning and academic performance in mathematics.

Again, a study by Agah (2015) on determinants of students' logical reasoning and mathematics achievement. The study employed an ex-post-facto research design. The study sample was 420 secondary students, 140 each of SSI, II, and III. Simple random sampling was used to draw seven (7) schools from 21 schools and four hundred and twenty (420) SS students from the 7 schools. The instrument for data collection was the mathematical reasoning test (MRT). The instrument is made up of ten (10) essay questions with five steps each to arrive at the correct answer. The instrument was validated by two mathematics educators and one measurement and evaluation lecturer. The test-retest method was used to determine the reliability of the instrument. The reliability coefficient yielded a Cronbach's alpha value of 0.84 using the Pearson product-moment correlation method. The data collected were analyzed using descriptive statistics, t-test statistics, and analysis of variance (ANOVA). The descriptive statistics were used to answer research questions, while the t-test and ANOVA were used to test the null hypothesis at a 0.05 level of significance. The study found that, among others, age and class level determine students' logical reasoning in mathematics. The study therefore recommended that age and class level should be given serious recognition in planning and organizing the mathematics curriculum.

H₃: Logical reasoning positively predicts mathematics achievement.

Conceptual Framework

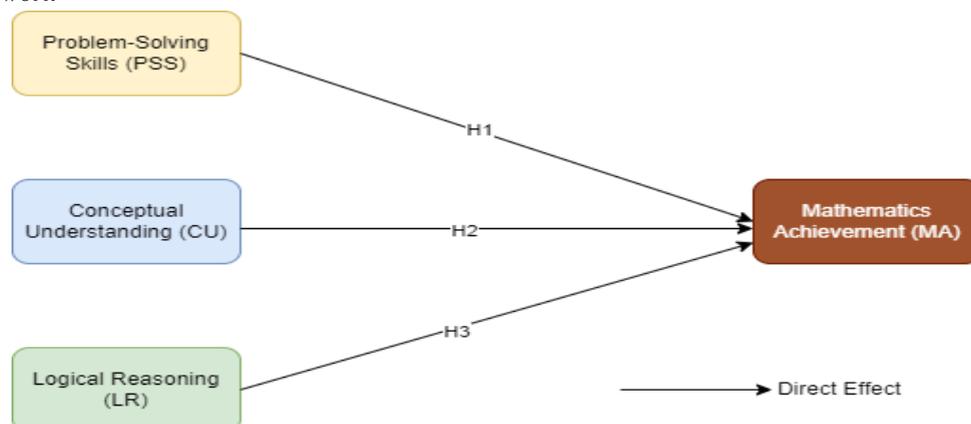


Figure 1: Conceptual Framework (Source: Author's Own Creation, 2026)

METHODOLOGY

This section presents the research design, population, sample size, and sampling techniques, questions and measures, and assessment of validity and reliability.

Research Design

Garage (2025) define research design as “*the overall plan or structure that guides the process of collecting, analyzing, and interpreting data in a study to address research questions or test hypotheses.*” This definition emphasizes that research design serves as a blueprint that ensures the research is logically structured and systematically executed. The study employed a descriptive research design utilizing a quantitative approach. Adeniran (2024), explain that when quantitative studies use large and representative samples, their results can be applied to wider contexts with a high degree of confidence. This makes quantitative research particularly valuable for policy development and educational decision-making.

Population

Liu et al., (2023) define population as “*the total number of items or individuals that have a common observable characteristic and form the universe from which a sample is selected.*” This focuses on the idea that the population represents the whole from which a researcher draws conclusions. The population of the study consists of senior high school students. The target population for the study comprises all students within Atwima Nwabiagya Municipality. The accessible population was 2528 students used for the study.

Sample Size and Sampling Techniques

The study employed a convenience sampling technique to select the schools. Again, simple random sampling techniques were used to select respondents. According to Ahmad and Khasawneh (2023), this ensures fairness and eliminates bias in the selection process. When every individual has an equal opportunity to be included, the likelihood of systematically favoring a particular subgroup within the population is minimized. This enhances the credibility and validity of the research findings, as the sample truly represents the diversity within the population. The sample size was calculated using Yarmane’s (1967) formula given below:

$$n = \frac{N}{1 + Ne^2}$$

Where n = sample size, N = population (2528), e = estimation error (0.05)

$$n = \frac{2528}{1 + 2528(0.05)^2}$$

$$n = 345$$

A sample of 345 students was used for the study.

Questions and Measures

Structured questionnaires with closed-ended questions were used to select the respondents for the study. Each question was validated using a five-point Likert scale from 1 “strongly agree” to 5 “strongly disagree”. The data were analyzed using structural equation modelling (SEM) in AMOS (ver.23), allowing us to determine whether Problem-Solving Skills (PSS) has a direct effect on Mathematics achievement (MA), Conceptual Understanding (CU) has a direct impact on Mathematics Achievement (MA), and Logical Reasoning (LR) has a direct impact on Mathematics Achievement (MA). Problem-Solving Skills (PSS) were adopted by (Asare et al., 2025). Each item was measured on a five-point Likert scale from 1 “strongly agree” to 5 “strongly disagree”. A higher score indicates a higher Cronbach’s alpha of 0.77, indicating high internal consistency. The present study’s Cronbach’s alpha value was 0.83 indicate high reliability. Conceptual Understanding (CU) was adopted from Sinaga et al., (2023). Each item was measured on a five-point Likert scale from 1 “strongly agree” to 5 “strongly disagree”. A higher score indicates a higher Cronbach’s alpha of 0.72, indicating high internal consistency. The present study’s Cronbach’s alpha value was 0.81, indicating high reliability. Logical Reasoning (LR) was adopted by Nawal Shirawia, et al., (2023). Each item was measured on a five-point Likert scale from 1 “strongly agree” to 5 “strongly disagree”. A higher score indicates a higher Cronbach’s alpha of 0.82, indicating high internal consistency. The present study’s Cronbach’s alpha value was 0.88, indicating high reliability.

Assessment of Validity and Reliability

To determine the validity of the instruments, the questionnaires were administered to experts in the field of mathematics education to ensure that the measurement items accurately measured. In Structural Equation Modeling (SEM), the assessment of the model’s variables’ validity and reliability is imperative to ensure a curated representation of the relationship between components. The results of the Exploratory Factor Analysis (EFA) presented in Table 2 revealed that the measurement items loaded appropriately on four components representing Problem Solving Skills (PSS), Conceptual Understanding (CU), Logical Reasoning (LR), and Mathematics Achievement (MA). The Rotated Component Matrix showed that items PSS1 (.878), PSS2 (.884), PSS3 (.869), and PSS4 (.831) loaded strongly on Component 1 (PSS). Similarly, CU1 (.879), CU2 (.848), CU3 (.840), and CU4 (.855) were highly loaded on Component 2 (CU). For Component 3 (LR), items LR1 (.836), LR2 (.851), LR3 (.853), and LR4 (.810) exhibited strong loadings, while Component 4 (MA) included MA1 (.820), MA2 (.821), MA3 (.806), and MA4 (.837), all exceeding the

recommended threshold of 0.70. These results indicate that all items are reliable indicators of their respective latent constructs (Cheung et al., 2024; Henseler et al., 2015; M R Ab Hamid et al., 2017).

Table 1: Exploratory Factor Analysis (EFA), KMO, and Bartlett's Test

Rotated Component Matrix				
Measurement Items	Component			
	1	2	3	4
PSS1	.878			
PSS2	.884			
PSS3	.869			
PSS4	.831			
CU1		.879		
CU2		.848		
CU3		.840		
CU4		.855		
LR1			.836	
LR2			.851	
LR3			.853	
LR4			.810	
MA1				.820
MA2				.821
MA3				.806
MA4				.837
KMO and Bartlett's Test				
Total Variance Explained (TVE)				
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.				.857
Bartlett's Test of Sphericity		Appox Chi-Square		3149.853
df	120			
sig.	.000			
Determinant			8.93E-005	

The Kaiser-Meyer-Olkin (KMO) value was 0.857, surpassing the recommended minimum of 0.60, which confirms that the sample size was adequate for factor analysis (Williams et al., 2020). The Bartlett's Test of Sphericity produced a Chi-Square value of 3149.853 with 120 degrees of freedom (df) and a significance level of 0.000, indicating that the correlation matrix was not an identity matrix and was therefore suitable for factor extraction. The determinant value (8.93E-005) was close to zero, further validating that the correlation matrix was appropriate for identifying underlying factors. The Total Variance Explained (TVE) further demonstrated that the four components jointly accounted for a substantial portion of the variance, signifying that the extracted factors capture the majority of the information contained in the observed variables. This confirms the appropriateness of the EFA model and the distinctiveness of each construct measured in the study (Gebremedhin et al., 2022; Marsh, 2022). In general, the EFA results confirmed that all items were well-loaded on their intended constructs, demonstrating strong construct validity and providing a reliable basis for conducting Confirmatory Factor Analysis (CFA) in the next stage of data analysis.

Table 2: Confirmatory Factor Analysis (CFA) and Model Fit Indices

Model fit: <i>CMIN=106.158; DF = 98; CMIN/DF = 1.083; TLI = .980; NFI = .829; CFI = .961; GFI = .961; SRMR = .021; RMSEA = .016; PClose = 1.000</i>	Std. Factor Loading
Problem solving Skills: CR = 0.910; CA = 0.831; AVE = 0.716	
I can apply different strategies to solve complex mathematical problems	.860
I can identify and understand the key elements of a mathematical problem before solving it	.878
I can check and evaluate my solutions to ensure they are accurate	.820
I can apply knowledge from one topic to solve problems in another area of mathematics	.826
Conceptual Understanding: CR= 0.900; CA = 0.821; AVE = 0.694	

I understand the underlying concepts behind mathematical formulas and procedures	.896
I can explain why a particular mathematical rule or method works	.816
I can connect new mathematical ideas to concepts I already know	.782
I can represent mathematical ideas in different forms (graphs, tables, or equations)	.832
Logical Reasoning: CR = 0.876; CA = 0.845; AVE = 0.638	
I can use logical steps to justify my answer in mathematics	.806
I can identify errors or inconsistencies in mathematical arguments	.831
I can make valid conclusions based on mathematical evidence	.810
I can reason through multi-step problems without guessing	.746
Mathematical Achievements: CR = 0.877; CA = 0.881; AVE = 0.640	
I consistently perform well in mathematics tests and assignments	.810
I feel confident when solving mathematical problems independently	.770
My mathematics grades affect my understanding and effort in the subject	.814
I can apply mathematical knowledge to solve real-life problems effectively	.806

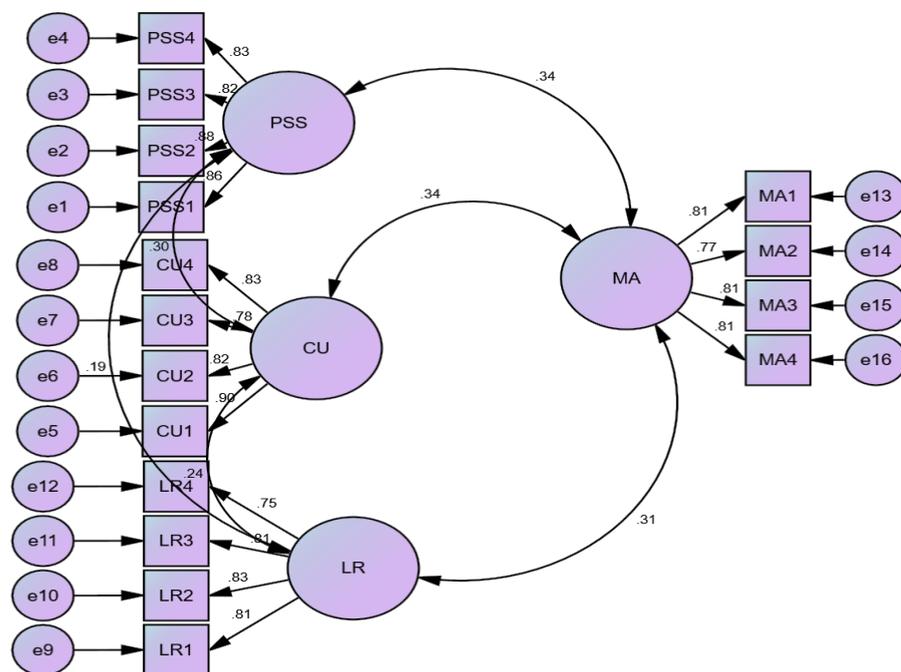


Figure 2: Confirmatory Factor Analysis (Source: Author’s Creation, 2026)

Discriminant validity was assessed using an add-in from AMOS (ver. 23), which involves comparing the square root of the AVE with the intercorrelated variables following Fornell and Larcker's (1981) criterion. The composite Reliability (CR), Average Variance Extracted (AVE), Maximum Shared Variance (MSV), and the maximum value of the correlation between each variable and any other variable (MaxR(H)) were calculated to evaluate the models’ variables, which include PSS (Problem Solving Skills), CU (Conceptual Understanding), LR (Logical Reasoning), and MA (Mathematics Achievement). Discriminant validity is achieved when the least square root of AVE exceeds the highest value of the intercorrelated variable. From Table 4, the least value of the square root of AVE was 0.799, and the highest value for the intercorrelated variable was 0.344. From this result, discriminant validity has been achieved.

Table 3: Discriminant Validity

Variables	CR	AVE	MSV	MaxR(H)	PSS	CU	LR	MA
PSS	0.910	0.716	0.119	0.912	0.846			
CU	0.900	0.694	0.117	0.908	0.300***	0.833		
LR	0.876	0.638	0.094	0.879	0.194**	0.236***	0.799	
MA	0.877	0.640	0.119	0.878	0.344***	0.343***	0.306***	0.800

Note: \sqrt{AVE} are bolded, *** means p-value significant at 1% (0.001), ** means ** p < 0.010

Key Findings

Table 4W: Path Summary

Direct Effect	Estimate	S.E.	C.R.	P-value
PSS→MA	.210	.057	3.684	***
CU→MA	.175	.053	3.302	***
LR→MA	.188	.056	3.357	***

Note: *** means p-value significant at 1 % (0.01)

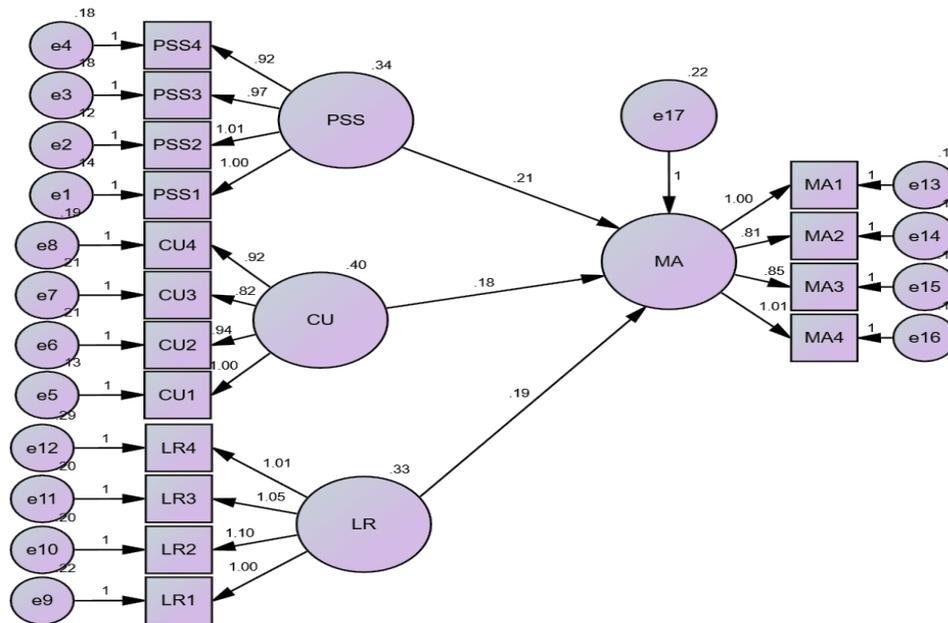


Figure 3: Path Diagram (Source: Authors' Own creation, 2026)

DISCUSSION OF RESULTS

H₁: Problem-solving skills positively predict mathematics achievement.

Research hypothesis 1 aimed to examine whether PSS has a positive effect on MA. Table 4 displays the results that support the research hypothesis. From table 4, PSS had a significant positive effect on mathematics achievement, with a p-value less than 1% ($\beta = 0.210, CR = 3.684$). The result further explains that PSS positively predicts MA with a percentage of 21.0. Therefore, we accept H_1 for the study.

H₂: Conceptual understanding positively predicts mathematics achievement.

Research hypothesis 2 aimed to examine whether CU has a positive effect on MA. Table 4 depicts the results that support the research hypothesis. From table 4, CU has a significant positive effect on mathematics achievement, with a p-value less than 1% ($\beta = 0.175, CR = 3.302$). These results further explain that CU positively predicts MA with a percentage of 17.5. Therefore, we accept H_2 for the study.

H₃: Logical reasoning positively predicts mathematics achievement.

Research hypothesis 3 aimed to examine whether LR has a positive effect on MA. Table 4 shows the results that support the research hypothesis. From table 4, LR has a significant positive effect on mathematics achievement, with a p-value less than 1% ($\beta = 0.188, CR = 3.357$). The results further explain that LR positively predicts MA with a percentage of 18.8. Therefore, we accept H_3 for the study.

DISCUSSIONS OF FINDINGS

The study found that Problem-Solving Skills (PSS) had a positive effect on Mathematics Achievement (MA). This finding is consistent with several recent studies, which emphasize that students with stronger problem-solving abilities tend to perform better in mathematics. These findings are in agreement with Adeoye and Jimoh (2023), problem-solving skills enable learners to analyze complex problems, develop creative solutions, and implement those solutions effectively. Also, the study found that problem-solving skills are essential in promoting innovation and creativity among learners. These skills enable one to identify and address challenges that arise in the process of innovation and creativity. Again, the study findings are in agreement with Sinaga et al., (2023), students' understanding of solving has a significant influence on mathematics learning results, with a p-value of .000.

The study also revealed that Conceptual Understanding (CU) had a positive effect on Mathematics Achievement (MA). These findings are in line with subjects with a visual mathematical thinking style, creating representations in the form of images, tables, or graphs for each indicator of understanding. Subjects with an analytical mathematical thinking style create representations in the form of verbal sentences or mathematical models that are rich in mathematical symbols and variables. While subjects with an integrated mathematical thinking style create representations in the form of verbal sentences, images, diagrams, mathematical symbols, tables, mathematical models, or graphs. The study concluded that while the type of mathematical thinking style- visual, analytical, or integrated-does not significantly influence the depths of students' understanding, it does shape the strategies they employ to construct that understanding. Again, the study findings are in agreement with Reni et al. (2025), ethnomathematics problem-based learning (PBL) enhances students' development of critical and logical thinking skills, as well as in connecting mathematical concepts with everyday contexts. The study found that there was a statistically significant difference between the experiential and control groups ($p < 0.05$), indicating that the ethnomathematics-based PBL model positively influences students' conceptual understanding.

Furthermore, the results indicated that Logical Reasoning (LR) positively influenced Mathematics Achievement (MA). The study findings are in line with Nawal et al., (2023), general average of the logical mathematical intelligence level for the fourth year students was in a high degree, when the arithmetic mean of the whole questionnaire was (3.71), standard deviation was (0.74). The marks of students' semester average were: Agree with a rate of 36.4%, strongly agree with a rate of 29.5%. The cumulative average was 72.7% for a good level, and 25% for a very good level. The study finally revealed that there is a significant effect between the logical mathematical intelligence of the female students and the overall academic achievements in mathematics. The findings are in agreement with Ramganes and Reddy (2021) as many as 67.8% of the variance could be predicted from students' Logical Reasoning on Academic Performance of students in mathematics, and significant correlation between logical reasoning and academic performance in mathematics. The findings are consistent with Agah (2015), among others that age and class level determine students' logical reasoning in mathematics.

CONCLUSION

In reconnoitering the relationship between Problem Solving Skills (PSS), Conceptual Understanding (CU), and Logical Reasoning (LR) on Mathematical Achievement (MA). Data were collected from 345 students in senior high schools within the Atwima Nwabiagya Municipality. The findings revealed that Problem-Solving Skills (PSS) predict Mathematical Achievement (MA). Also, Conceptual Understanding predicts Mathematical Achievement (MA), and Logical Reasoning (LR) predicts Mathematical Achievement (MA). These findings hold important implications not only for students in the selected senior high schools in Ghana but also for higher education institutions aiming to enhance student mathematics achievement through Problem Solving skills (PSS), Conceptual Understanding (CU), and Logical Reasoning (LR). These findings suggest that strengthening students' abilities to analyse problems, apply appropriate strategies, and evaluate solutions can lead to improved mathematics performance. This means mathematics teachers and curriculum planners should integrate more problem-solving tasks, discovery-based activities, and real-world applications into instructional strategies. Schools may also need to prioritize training teachers in problem-solving pedagogy since improved PSS directly enhances students' achievement. Also, the findings show that result shows that when students deeply understand mathematical concepts, they can connect ideas, transfer knowledge, and adapt to new problem types. This underscores the need for concept-based instruction rather than procedural teaching alone. Curriculum designers must ensure that learning materials emphasize concept development, visualization, and reasoning so that students build a strong foundational understanding that can support long-term mathematics success. Again, Logical reasoning enables students to make valid conclusions, identify patterns, justify steps, and think critically, skills that are essential in many mathematics topics such as algebra, geometry, and statistics. This means teachers should intentionally incorporate reasoning activities, such as deductive tasks, justification exercises, and inquiry-based discussions, into daily teaching. The result further implies that developing students' reasoning abilities is not only beneficial for mathematics but also for broader cognitive development, including decision-making and analytical thinking. Mathematics instruction should prioritize deep conceptual understanding rather than focusing solely on procedures. Teachers should use visual models, manipulatives, concept mapping, and explanations that help students connect ideas and understand the underlying principles of mathematics concepts.

LIMITATIONS AND SUGGESTIONS FOR FURTHER STUDIES

Despite the methodological rigor of the study, several limitations must be acknowledged. First, the use of a descriptive quantitative design limits the ability to draw causal conclusions about the relationships between Problem-Solving Skills (PSS), Conceptual Understanding (CU), Logical Reasoning (LR), and Mathematics Achievement (MA). Although Structural Equation Modeling (SEM) provided strong statistical evidence of associations, the design cannot fully explain why or how these relationships develop over time. Second, the study relied solely on self-reported questionnaires, which may introduce social desirability bias and limit the accuracy of students' responses, especially regarding their mathematics abilities. Additionally, the use of convenience sampling to select schools restricts the representativeness of the larger population, even though simple random sampling was applied at the

student level. The accessible sample of 345 students, although adequate for SEM, may not fully capture the diversity of all senior high schools in the Atwima Nwabiagya Municipality. Furthermore, the study focused on only three predictors (PSS, CU, and LR), excluding other crucial variables such as motivation, teacher effectiveness, classroom environment, socioeconomic background, and curriculum quality, which may also significantly influence mathematics achievement.

Future research should consider adopting a mixed-method or longitudinal research design to provide deeper insights into the causal mechanisms underlying the relationships identified in this study. Qualitative methods, such as interviews or classroom observations, could complement quantitative data and help explain how students develop reasoning and problem-solving competencies in real learning contexts. Longitudinal designs can also track students' growth in these skills over time, offering more robust evidence on directionality. Additionally, expanding the study to include larger and more diverse school samples across multiple districts would enhance generalizability and allow for comparative analysis between urban, peri-urban, and rural settings. Further studies may also incorporate additional variables, such as teacher instructional practices, student motivation, parental involvement, and availability of learning resources, to build a more holistic model of mathematics achievement. Researchers could also explore moderating or mediating effects, such as whether conceptual understanding mediates the relationship between problem-solving skills and mathematics achievement, or whether gender, school type, or socioeconomic status moderates the effects of LR, CU, and PSS on MA. These expansions would strengthen theoretical development and support more effective educational interventions.

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